#### **Degenerate Transportation Problem**

In a transportation problem, if a basic feasible solution with m origins and n destinations has less than m + n - 1 positive  $X_{ij}$  i.e. occupied cells, then the problem is said to be a **degenerate transportation problem**. The degeneracy problem does not cause any serious difficulty, but it can cause computational problem wile determining the optimal minimum solution.

Therefore it is important to identify a degenerate problem as early as beginning and take the necessary action to avoid any computational difficulty. The degeneracy can be identified through the following results:

"In a transportation problem, a degenerate basic feasible solution exists if and only if some partial sum of supply (row) is equal to a partial sum of demand (column). For example the following transportation problem is degenerate. Because in this problem

 $\begin{array}{l} a_1 = 400 = b_1 \\ a_2 + a_3 = 900 = b_2 + {}_{b3} \end{array}$ 

Warehouses								
w1	w2	w3	Supply (a <sub>i</sub> )					
20	17	25						
			400					
10	10	20						
10	10	20	500					
0	0	0						
0	0	0	400					
400	400	500	1300					
	20 10 0	w1 w2   20 17   10 10   0 0	w1 w2 w3   20 17 25   10 10 20   0 0 0					

There is a technique called perturbation, which helps to solve the degenerate problems.

### **Perturbation Technique:**

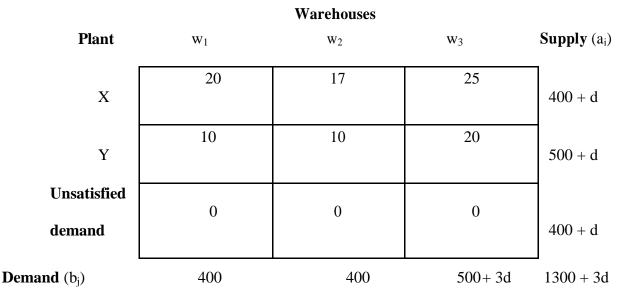
The degeneracy of the transportation problem can be avoided if we ensure that no partial sum of ai (supply) and bj (demand) is equal.

We set up a new problem where

$a_i = a_i + d$	i = 1, 2,, m
$\mathbf{b}_{\mathbf{j}} = \mathbf{b}_{\mathbf{j}}$	j = 1, 2,, n -1
$\dot{b_n} = \dot{b_n} + m_d$	d > 0

This modified problem is constructed in such a way that no partial sum of  $a_i$  is equal to the  $b_j$ . Once the problem is solved, we substitute d = 0 leading to optimum solution of the original problem. **Example:** 

# Consider the above problem



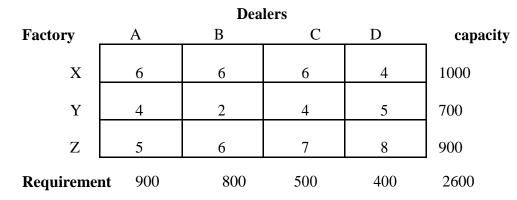
Now this modified problem can be solved by using any of the three methods viz. North-west Corner, Least Cost, or VAM.

# **Transportation Problem Maximization**

There are certain types of transportation problem where the objective function is to be maximized instead of minimized. These kinds of problems can be solved by converting the maximization problem into minimization problem. The conversion of maximization into minimization is done by subtracting the unit costs from the highest unit cost of the table.

## Example

A company has three factories located in three cities viz. X, Y, Z. These factories supplies consignments to four dealers viz. A, B, C and D. The dealers are spread all over the country. The production capacity of these factories is 1000, 700 and 900 units per month respectively. The net return per unit product is given in the following table.

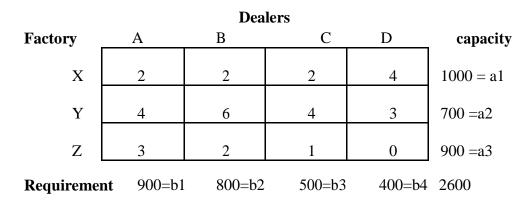


Determine a suitable allocation to maximize the total return.

This is a maximization problem. Hence first we have to convert this in to minimization problem. The conversion of maximization into minimization is done by subtracting the unit cost of the table from

the highest unit cost.

Look the table, here 8 is the highest unit cost. So, subtract all the unit cost from the 8, and then we get the revised minimization transportation table, which is given below.



Now we can solve the problem as a minimization problem.

The problem here is degenerate, since the partial sum of  $a_1=b_2+b_3$  or  $a_3=b_3$ . So consider the corresponding perturbed problem, which is shown below.

Dealers								
Factory	А	В	С	D	capacity			
Х	2	2	2	4	1000+d			
Y	4	6	4	3	700+d			
Z	3	2	1	0	900+d			
Requiremen	nt 900	800	500	400+3d	2600+3d			

First we have to find out the basic feasible solution. The basic feasible solution by lest cost method is  $x_{11}=100+d$ ,  $x_{22}=700-d$ ,  $x_{23}=2d$ ,  $x_{33}=500-2d$  and  $x_{34}=400+3d$ .

Once if the basic feasible solution is found, next we have to determine the optimum solution using MODI (Modified Distribution Method) method. By using this method we obtain

$$u_1+v_1=2$$
  $u_1+v_2=2$   $u_2+v_2=6$   
 $u_2+v_3=4$   $u_3+v_3=1$   $u_3+v_4=0$ 

Taking  $u_1=0$  arbitrarily we obtain  $u_1=0, u_2=4, u_3=1$  and  $v_1=2, v_2=3, v_3=0$  On verifying the condition of optimality, we know that

 $C_{12}$ -u<sub>1</sub>-v<sub>2</sub> < 0 and  $C_{32}$ -u<sub>3</sub>-v<sub>2</sub> < 0

So, we allocate  $x_{12}$ =700-d and make readjustment in some of the other basic variables.

The revised values are:

 $\begin{array}{c} x_{11} = 200 + d, \ x_{12} = 800, \ x_{21} = 700 - d, \ x_{23} = 2d, \ x_{33} = 500 - 3d, \ and \ x_{34} = 400 + 3d \\ \\ u_1 + v_1 = 2 & u_1 + v_2 = 2 & u_2 + v_1 = 4 \\ u_2 + v_3 = 4 & u_3 + v_3 = 1 & u_3 + v_4 = 0 \\ \\ Taking \ u_1 = 0 \ arbitrarily \ we \ obtain \\ u_1 = 0, \ u_2 = 2, \ u_3 = -1 \\ v_1 = 2, \ v_2 = 2, \ v_3 = 2, \ v_4 = 1 \\ \\ Now, \ the \ optimality \ condition \ is \ satisfied. \end{array}$ 

Finally, taking d=0 the optimum solution of the transportation problem is  $X_{11}$ =200,  $x_{12}$ =800,  $x_{21}$ =700,  $x_{33}$ =500 and  $x_{34}$ =400

Thus, the maximum return is:

6\*200 + 6\*800 + 4\*700 + 7\*500 + 8\*400 = 15500